Pin risk management

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Pin risk arises when the underlying futures contract settles at or very near a strike price, creating uncertainty over option exercise decisions (short leg exercise). Consider a portfolio consisting of futures, calls, and puts (under same expiry). Suppose the underlying futures price closes at a strike—referred to as the *pin strike*. Excluding the option pair at this strike, the total delta of the remaining portfolio is denoted by q_f. At the pin strike, let q_c and q_p denote the quantities of call and put options held. These positions are classified as long or short. Define q_L as the total quantity of long contracts (calls or puts), q_S as the total quantity of short contracts, δ_L as the delta of each long contract (+1 for long calls, -1 for long puts), δ_S as the delta of each short contract (opposite sign to δ_L). For example, long 20 calls and short 50 puts gives $q_L = 20$, $q_S = 50$, $\delta_L = 1$, $\delta_S = -1$. Let $x \in [0, q_L]$ be the number of long contracts exercised, and $y \in [0, q_S]$ the number of short contracts exercised by the counterparty. The net delta contribution from the pin strike position is $\delta_{total} = q_f + x\delta_L - y\delta_S$. To minimise post-maturity risk exposure, we solve

$$\min_{0 \le x \le q_L} \mathcal{R}(x),$$

where the risk utility function is defined as

$$\mathcal{R}(x) = \mathbb{E}_{y \sim U(0,q_S)}\left[(q_f + x\delta_L - y\delta_S)^2\right].$$

With $y \sim U(0, q_S)$ and density $1/q_S$, expanding and taking expectations gives

$$\mathcal{R}(x) = (q_f + x\delta_L)^2 - (q_f + x\delta_L)\delta_S q_S + \frac{\delta_S^2 q_S^2}{3}.$$

Expanding in x:

$$\mathcal{R}(x) = \delta_L^2 x^2 + \delta_L(2q_f - \delta_S q_S)x + q_f^2 - \delta_S q_S q_f + \frac{\delta_S^2 q_S^2}{3}.$$

Since $\delta_L^2 = \delta_S^2 = 1$ and $\delta_L\delta_S = -1$,

$$\mathcal{R}(x) = x^2 + \delta_L(2q_f - \delta_S q_S)x + q_f^2 - \delta_S q_S q_f + \frac{q_S^2}{3}.$$

Setting $\mathcal{R}'(x) = 0$:

$$2x + \delta_L(2q_f - \delta_S q_S) = 0 \quad \Rightarrow \quad x^* = -\delta_L q_f + \frac{\delta_L \delta_S q_S}{2}.$$

Using $\delta_L\delta_S = -1$,

$$x^* = -\delta_L q_f - \frac{q_S}{2}.$$

Clipping to $[0, q_L]$ yields the feasible optimum:

$$\boxed{x^* = \max\Big(\min\big(-\delta_L q_f - \frac{q_S}{2},\ q_L\big),\ 0\Big).}$$

Table 1: Examples

call.pos	put.pos	future.pos (q_f)	δ_S	δ_L	q_S	q_L	x^*
100	-100	0	-1	1	100	100	0
100	-100	-80	-1	1	100	100	30
100	-100	-40	-1	1	100	100	0
-100	100	-40	1	-1	100	100	0

The solution for a long call—long put position at the pin strike is straightforward and is not discussed herein.